Student Notes

Student Session Topic: Limits and Continuity

The AP Calculus exams include multiple-choice and free-response questions on limits and continuity. There are on the order of 4 – 6 multiple choice questions on these topics each year. The skills needed are listed below. None of these questions require you to "prove" or "justify" a limit or use the $\delta-\varepsilon$ definition of limit; they only ask you to find a limit.

What you should know how to do:

- Compute limits algebraically
- Determine limits from a graph
- Know the relationship between limits and asymptotes – An asymptote is the graphical manifestation of a limit at infinity or a limit equal to infinity.
- Recognize the limit definition of derivative – be able to identify the function involved and the point at which the derivative is evaluated. (See multiple choice 10 ,11)
- Discuss continuity algebraically and graphically and know its relation to limit.
- BC Only: L'Hôpital's Rule
- BC only: Limits associated with the logistic equation

Dominance

While not asked directly the concept of "dominance" is often the quickest (and best) way to find a limit as $x \to \pm\infty$. You should know that *exponential functions always increase faster than polynomial and power functions* and *polynomial functions always increase faster than logarithmic functions*. Between two exponential or two polynomial functions the one with the larger exponent dominates as $x \to \pm\infty$. (See multiple-choice 5 and 1998 AB 2(a))

Example: find $\lim\limits_{x\to\infty}\dfrac{\ln\left(x^5\right)}{x^{0.02}} =$

Answer: $\lim\limits_{x\to\infty}\dfrac{\ln\left(x^5\right)}{x^{0.02}} = 0$. Eventually, $x^{0.02}$ will become larger than $\ln\left(x^5\right)$. It's a fun calculator question to find the maximum point on this graph $\left(5.185\times10^{21}, 91.970\right)$ and its point of inflection $\left(1.382\times10^{22}, 91.952\right)$. Notice the numbers. The point of inflection is 2.67 times as far from the origin as the maximum point, yet the drop in *y*-values is only 0.018.

Student Notes

Continuity

- A function is continuous at a point *x = a* if and only if $f\left(a\right)$ is finite, $\lim_{x \to a} f\left(x\right)$ is finite and $\lim_{x \to a} f\left(x\right) = f\left(a\right)$ (the limit equals the value).
- A function is continuous on an interval if it is continuous at every point of the interval.
- Differentiability implies continuity: If a function is differentiable at a point (at every point on an interval), then it is continuous at that point (on that interval). The converse is false: continuous function may not be differentiable. Example $y = |x|$ at (0, 0).

Multiple-choice questions

a. Questions 1 – 4 Graphical

b. Questions 5 – 7 Algebraic

c. Questions 8 – 9 Continuity and differentiability

d. Questions 10 – 11 Definition of derivative

e. Questions 12 – 14 BC Only L'Hôpital's Rule & logistic equation.

Answers 1 B, 2 A, 3 D, 4 A, 5 C, 6 B, 7 A, 8 E, 9 D, 10 C, 11 B, 12 C, 13 E, 14 B

WATCH and LISTEN to the multiple-choice questions being solved.

Go to http://tinyurl.com/NMSI-Math-2 Click on the "Full Screen" arrow. Then click anywhere on the page to see and hear from that point on. Click to go back anytime.

Free-response questions

AB and BC

1998 AB2/BC2 (a), (c),

In (a) you need only write the answer. The one limit is either ∞ or DNE (either answer is correct). The minimum value is found in (b) and then the limits must be interpreted to find the range of the function in (c).

Student Notes

2003 AB 6

Limits are necessary to explain continuity in (a) and necessary to find one of the equations necessary in (c).

For BC only

2001 BC 6 (b)

Simplifying the expression in (b) makes the limit easy to find. (Similar to 2007 BC 6 (b))

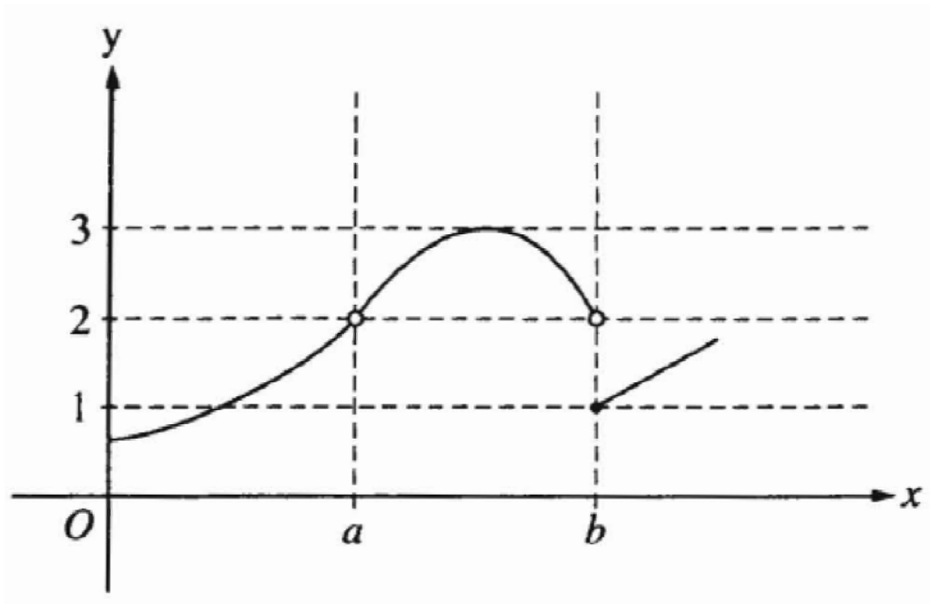
2004 BC 5 (Logistic equation)

You do not need to solve the logistic differential equation to answer this question. The equation has horizontal asymptotes at *y* = 0 and *y* = 12, because that's where $\frac{dP}{dt} = 0$. The initial conditions are below and above the asymptote so as $x \to \infty$, the function must approach the asymptote.

Limits and Continuity

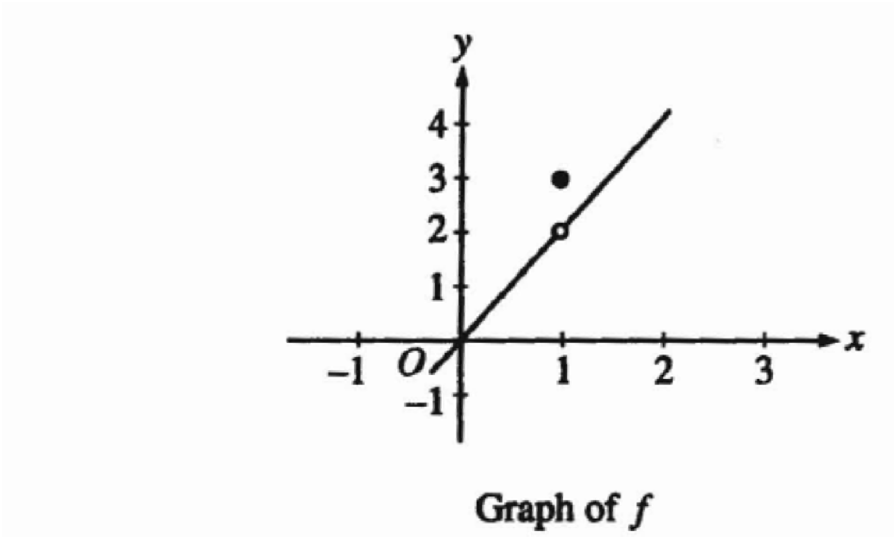
Multiple Choice
Identify the choice that best completes the statement or answers the question.

1.



The graph of the function f is shown in the figure above. Which of the following statements about f is true?

a. $\lim_{x \to a} f(x) = \lim_{x \to b} f(x)$
b. $\lim_{x \to a} f(x) = 2$
c. $\lim_{x \to b} f(x) = 2$
d. $\lim_{x \to b} f(x) = 1$
e. $\lim_{x \to a} f(x)$ does not exist

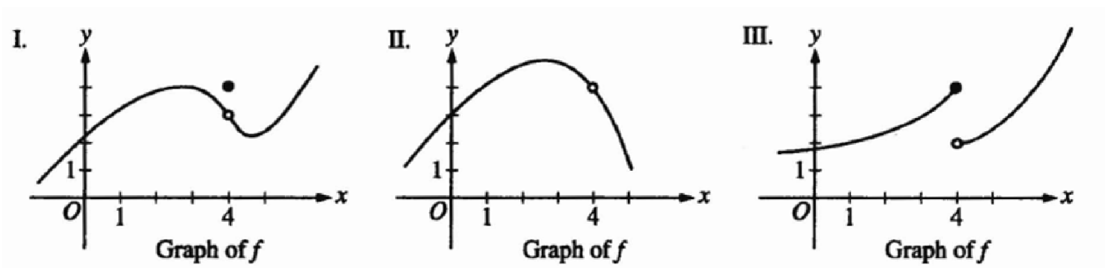
2. 

Graph of f

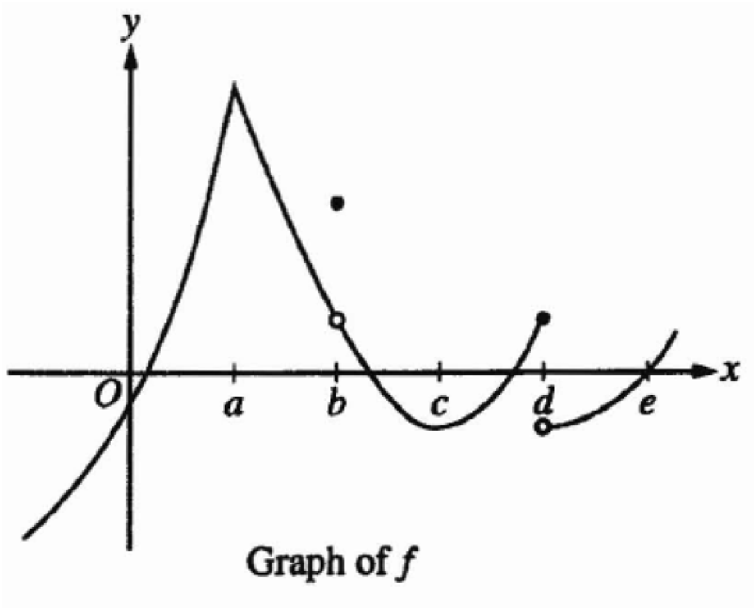
The graph of the function f is shown in the figure above. The value of $\lim_{x \to 1} \sin\left(f(x)\right)$ is

a. 0.909
b. 0.841
c. 0.141
d. -0.416
e. nonexistent

3. For which of the following does $\lim_{x \to 4} f(x)$ exist?



a. I only
b. II only
c. III only
d. I and II only
e. I and III only

4. 

The graph of a function *f* is shown above. At which value of *x* is *f* continuous, but not differentiable?

a. *a*
b. *b*
c. *c*
d. *d*
e. *e*

5. $\lim\limits_{x \to \infty} \dfrac{x^3 - 2x^2 + 3x - 4}{4x^3 - 3x^2 + 2x - 1} =$

a. 4
b. 1
c. $\dfrac{1}{4}$
d. 0
e. -1

6. If $a \neq 0$, then $\lim\limits_{x \to a} \dfrac{x^2 - a^2}{x^4 - a^4}$ is

a. $\dfrac{1}{a^2}$
b. $\dfrac{1}{2a^2}$
c. $\dfrac{1}{6a^2}$
d. 0
e. nonexistent

7. Let f be the function given by $f(x) = \dfrac{(x-1)\left(x^2-4\right)}{x^2-a}$. For what positive values of a is f continuous for all real numbers x?

a. None
b. 1 only
c. 2 only
d. 4 only
e. 1 and 4 only

8. If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \le 2 \\ x^2 \ln 2 & \text{for } 2 < x \le 4, \end{cases}$ the $\lim\limits_{x\to 2} f(x)$ is

a. $\ln 2$
b. $\ln 8$
c. $\ln 16$
d. 4
e. nonexistent

9.

$$f(x) = \begin{cases} x+2 & \text{if } x \le 3 \\ 4x-7 & \text{if } x > 3 \end{cases}$$

Let f be the function given above. Which of the following statements are true about f?

I. $\lim\limits_{x\to 3} f(x)$ exists
II. f is continuous at $x = 3$.
III. f is differentiable at $x = 3$.

a. None
b. I only
c. II only
d. I and II only
e. I, II, and III

10. Let f be a function such that $\lim\limits_{h\to 0} \dfrac{f(2+h)-f(2)}{h} = 5$. Which of the following must be true?

I. f is continuous at x = 2.
II. f is differentiable at x = 2.
III. The derivative of f is continuous at x = 2.

a. I only
b. II only
c. I and II only
d. I and III only
e. II and III only

11. $\lim\limits_{h \to 0} \dfrac{e^h - 1}{2h}$ is

a. 0
b. $\dfrac{1}{2}$
c. 1
d. e
e. nonexistent

12. $\lim\limits_{x \to 1} \dfrac{\int_1^x e^{t^2}\, dt}{x^2 - 1}$ is

a. 0
b. 1
c. $\dfrac{e}{2}$
d. e
e. nonexistent

13. The population $P(t)$ of a species satisfies the logistic differential equation $\dfrac{dP}{dt} = P\left(2 - \dfrac{P}{5000}\right)$, where the initial population $P(0) = 3{,}000$ and t is the time in years. What is $\lim\limits_{t \to \infty} P(t)$?

a. 2,500
b. 3,000
c. 4,200
d. 5,000
e. 10,000

14. The number of moose in a national park is modeled by the function M that satisfies the logistic differential equation $\dfrac{dM}{dt} = 0.6M\left(1 - \dfrac{M}{200}\right)$, where t is the time in years and M(0) =50. What is $\lim\limits_{t \to \infty} M(t)$?

a. 50
b. 200
c. 500
d. 1000
e. 2000

2. Let f be the function given by $f(x) = 2xe^{2x}$.

   (a) Find $\lim\limits_{x \to -\infty} f(x)$ and $\lim\limits_{x \to \infty} f(x)$.

   (b) Find the absolute minimum value of f. Justify that your answer is an absolute minimum.

   (c) What is the range of f?

   (d) Consider the family of functions defined by $y = bxe^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of b.

GO ON TO THE NEXT PAGE

1998 Calculus AB Scoring Guidelines

2. Let f be the function given by $f(x) = 2xe^{2x}$.
   (a) Find $\lim_{x\to-\infty} f(x)$ and $\lim_{x\to\infty} f(x)$.
   (b) Find the absolute minimum value of f. Justify that your answer is an absolute minimum.
   (c) What is the range of f?
   (d) Consider the family of functions defined by $y = bxe^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of b.

(a) $\lim_{x\to-\infty} 2xe^{2x} = 0$

$\lim_{x\to\infty} 2xe^{2x} = \infty$ or DNE

$$\mathbf{2}\begin{cases} 1: & 0 \text{ as } x \to -\infty \\ 1: & \infty \text{ or DNE as } x \to \infty \end{cases}$$

(b) $f'(x) = 2e^{2x} + 2x \cdot 2 \cdot e^{2x} = 2e^{2x}(1 + 2x) = 0$

if $x = -1/2$

$f(-1/2) = -1/e$ or -0.368 or -0.367

$-1/e$ is an absolute minimum value because:

(i) $f'(x) < 0$ for all $x < -1/2$ and
$f'(x) > 0$ for all $x > -1/2$

–or–

(ii) $f'(x)$ $\quad -\quad\quad +$

$-1/2$

and $x = -1/2$ is the only critical number

$$\mathbf{3}\begin{cases} 1: & \text{solves } f'(x) = 0 \\ 1: & \text{evaluates } f \text{ at student's critical point} \\ & 0/1 \text{ if not local minimum from} \\ & \text{student's derivative} \\ 1: & \text{justifies absolute minimum value} \\ & 0/1 \text{ for a local argument} \\ & 0/1 \text{ without explicit symbolic} \\ & \text{derivative} \end{cases}$$

Note: 0/3 if no absolute minimum based on student's derivative

(c) Range of $f = [-1/e, \infty)$
or $[-0.367, \infty)$
or $[-0.368, \infty)$

1: answer

Note: must include the left–hand endpoint; exclude the right–hand "endpoint"

(d) $y' = be^{bx} + b^2xe^{bx} = be^{bx}(1 + bx) = 0$

if $x = -1/b$

At $x = -1/b$, $y = -1/e$

y has an absolute minimum value of $-1/e$ for all nonzero b

$$\mathbf{3}\begin{cases} 1: & \text{sets } y' = be^{bx}(1 + bx) = 0 \\ 1: & \text{solves student's } y' = 0 \\ 1: & \text{evaluates } y \text{ at a critical number} \\ & \text{and gets a value independent of } b \end{cases}$$

Note: 0/3 if only considering specific values of b

6. Let f be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \le x \le 3 \\ 5 - x & \text{for } 3 < x \le 5. \end{cases}$$

(a) Is f continuous at $x = 3$? Explain why or why not.

(b) Find the average value of $f(x)$ on the closed interval $0 \le x \le 5$.

(c) Suppose the function g is defined by

$$g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \le x \le 3 \\ mx + 2 & \text{for } 3 < x \le 5, \end{cases}$$

where k and m are constants. If g is differentiable at $x = 3$, what are the values of k and m ?

END OF EXAMINATION

Question 6

Let f be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \le x \le 3 \\ 5 - x & \text{for } 3 < x \le 5. \end{cases}$$

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$$g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \le x \le 3 \\ mx + 2 & \text{for } 3 < x \le 5, \end{cases}$$

where k and m are constants. If g is differentiable at $x = 3$, what are the values of k and m?

(a) f is continuous at $x = 3$ because

$\lim\limits_{x\to 3^-} f(x) = \lim\limits_{x \to 3^+} f(x) = 2.$

Therefore, $\lim\limits_{x\to 3} f(x) = 2 = f(3).$

$2: \begin{cases} 1: \text{answers “yes” and equates the values of the left- and right-hand limits} \\ 1: \text{explanation involving limits} \end{cases}$

(b) $$\int_0^5 f(x)\,dx = \int_0^3 f(x)\,dx + \int_3^5 f(x)\,dx$$
$$= \frac{2}{3}(x+1)^{3/2}\Big|_0^3 + \left(5x - \frac{1}{2}x^2\right)\Big|_3^5$$
$$= \left(\frac{16}{3} - \frac{2}{3}\right) + \left(\frac{25}{2} - \frac{21}{2}\right) = \frac{20}{3}$$

Average value: $\frac{1}{5}\int_0^5 f(x)\,dx = \frac{4}{3}$

$4: \begin{cases} 1: k\int_0^3 f(x)\,dx + k\int_3^5 f(x)\,dx \\ \quad (\text{where } k \ne 0) \\ 1: \text{antiderivative of } \sqrt{x+1} \\ 1: \text{antiderivative of } 5 - x \\ 1: \text{evaluation and answer} \end{cases}$

(c) Since g is continuous at $x = 3$, $2k = 3m + 2$.

$$g'(x) = \begin{cases} \dfrac{k}{2\sqrt{x+1}} & \text{for } 0 < x < 3 \\ m & \text{for } 3 < x < 5 \end{cases}$$

$\lim\limits_{x\to 3^-} g'(x) = \frac{k}{4}$ and $\lim\limits_{x\to 3^+} g'(x) = m$

Since these two limits exist and g is differentiable at $x = 3$, the two limits are equal. Thus $\frac{k}{4} = m$.

$8m = 3m + 2$; $m = \frac{2}{5}$ and $k = \frac{8}{5}$

$3: \begin{cases} 1: 2k = 3m + 2 \\ 1: \frac{k}{4} = m \\ 1: \text{values for } k \text{ and } m \end{cases}$

6. A function f is defined by

$$f(x) = \frac{1}{3} + \frac{2}{3^2}x + \frac{3}{3^3}x^2 + \cdots + \frac{n+1}{3^{n+1}}x^n + \cdots$$

for all x in the interval of convergence of the given power series.

(a) Find the interval of convergence for this power series. Show the work that leads to your answer.

(b) Find $\lim\limits_{x\to 0} \dfrac{f(x) - \frac{1}{3}}{x}$.

(c) Write the first three nonzero terms and the general term for an infinite series that represents $\int_0^1 f(x)\,dx$.

(d) Find the sum of the series determined in part (c).

END OF EXAMINATION

Question 6

A function f is defined by

$$f(x) = \frac{1}{3} + \frac{2}{3^2}x + \frac{3}{3^3}x^2 + \cdots + \frac{n+1}{3^{n+1}}x^n + \cdots$$

for all x in the interval of convergence of the given power series.

(a) Find the interval of convergence for this power series. Show the work that leads to your answer.

(b) Find $\lim\limits_{x \to 0} \dfrac{f(x) - \frac{1}{3}}{x}$.

(c) Write the first three nonzero terms and the general term for an infinite series that represents $\int_0^1 f(x)\,dx$.

(d) Find the sum of the series determined in part (c).

(a) $$\lim_{n\to\infty}\left|\frac{\dfrac{(n+2)x^{n+1}}{3^{n+2}}}{\dfrac{(n+1)x^n}{3^{n+1}}}\right| = \lim_{n\to\infty}\left|\frac{(n+2)}{(n+1)}\frac{x}{3}\right| = \left|\frac{x}{3}\right| < 1$$

At $x = -3$, the series is $\sum\limits_{n=0}^{\infty}(-1)^n\dfrac{n+1}{3}$, which diverges.

At $x = 3$, the series is $\sum\limits_{n=0}^{\infty}\dfrac{n+1}{3}$, which diverges.

Therefore, the interval of convergence is $-3 < x < 3$.

4 :
- 1 : sets up ratio test
- 1 : computes limit
- 1 : conclusion of ratio test
- 1 : endpoint conclusion

(b) $$\lim_{x\to 0}\frac{f(x)-\frac{1}{3}}{x} = \lim_{x\to 0}\left(\frac{2}{3^2} + \frac{3}{3^3}x + \frac{4}{3^4}x^2 + \cdots\right) = \frac{2}{9}$$

1 : answer

(c) $$\int_0^1 f(x)\,dx = \int_0^1\left(\frac{1}{3} + \frac{2}{3^2}x + \frac{3}{3^3}x^2 + \cdots + \frac{n+1}{3^{n+1}}x^n + \cdots\right)dx$$
$$= \left(\frac{1}{3}x + \frac{1}{3^2}x^2 + \frac{1}{3^3}x^3 + \cdots + \frac{1}{3^{n+1}}x^{n+1} + \cdots\right)\Bigg|_{x=0}^{x=1}$$
$$= \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \cdots + \frac{1}{3^{n+1}} + \cdots$$

3 :
- 1 : antidifferentiation of series
- 1 : first three terms for definite integral series
- 1 : general term

(d) The series representing $\int_0^1 f(x)\,dx$ is a geometric series.

Therefore, $\int_0^1 f(x)\,dx = \dfrac{\frac{1}{3}}{1-\frac{1}{3}} = \dfrac{1}{2}$.

1 : answer

6. Let f be the function given by $f(x) = e^{-x^2}$.

(a) Write the first four nonzero terms and the general term of the Taylor series for f about $x = 0$.

(b) Use your answer to part (a) to find $\lim\limits_{x \to 0} \dfrac{1 - x^2 - f(x)}{x^4}$.

(c) Write the first four nonzero terms of the Taylor series for $\int_0^x e^{-t^2}\,dt$ about $x = 0$. Use the first two terms of your answer to estimate $\int_0^{1/2} e^{-t^2}\,dt$.

(d) Explain why the estimate found in part (c) differs from the actual value of $\int_0^{1/2} e^{-t^2}\,dt$ by less than $\dfrac{1}{200}$.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF EXAM

Question 6

Let f be the function given by $f(x) = e^{-x^2}$.

(a) Write the first four nonzero terms and the general term of the Taylor series for f about $x = 0$.

(b) Use your answer to part (a) to find $\lim\limits_{x\to 0} \dfrac{1 - x^2 - f(x)}{x^4}$.

(c) Write the first four nonzero terms of the Taylor series for $\int_0^x e^{-t^2}\,dt$ about $x = 0$. Use the first two terms of your answer to estimate $\int_0^{1/2} e^{-t^2}\,dt$.

(d) Explain why the estimate found in part (c) differs from the actual value of $\int_0^{1/2} e^{-t^2}\,dt$ by less than $\dfrac{1}{200}$.

(a) $e^{-x^2} = 1 + \dfrac{(-x^2)}{1!} + \dfrac{(-x^2)^2}{2!} + \dfrac{(-x^2)^3}{3!} + \cdots + \dfrac{(-x^2)^n}{n!} + \cdots$ $= 1 - x^2 + \dfrac{x^4}{2} - \dfrac{x^6}{6} + \cdots + \dfrac{(-1)^n x^{2n}}{n!} + \cdots$	$3: \begin{cases} 1: \text{two of } 1, -x^2, \frac{x^4}{2}, -\frac{x^6}{6} \\ 1: \text{remaining terms} \\ 1: \text{general term} \end{cases}$
(b) $\dfrac{1 - x^2 - f(x)}{x^4} = -\dfrac{1}{2} + \dfrac{x^2}{6} + \sum\limits_{n=4}^{\infty} \dfrac{(-1)^{n+1} x^{2n-4}}{n!}$ Thus, $\lim\limits_{x\to 0}\left(\dfrac{1 - x^2 - f(x)}{x^4}\right) = -\dfrac{1}{2}$.	1 : answer
(c) $\int_0^x e^{-t^2}\,dt = \int_0^x \left(1 - t^2 + \dfrac{t^4}{2} - \dfrac{t^6}{6} + \cdots + \dfrac{(-1)^n t^{2n}}{n!} + \cdots\right) dt$ $= x - \dfrac{x^3}{3} + \dfrac{x^5}{10} - \dfrac{x^7}{42} + \cdots$ Using the first two terms of this series, we estimate that $\int_0^{1/2} e^{-t^2}\,dt \approx \dfrac{1}{2} - \left(\dfrac{1}{3}\right)\left(\dfrac{1}{8}\right) = \dfrac{11}{24}$.	$3: \begin{cases} 1: \text{two terms} \\ 1: \text{remaining terms} \\ 1: \text{estimate} \end{cases}$
(d) $\left\lvert \int_0^{1/2} e^{-t^2}\,dt - \dfrac{11}{24} \right\rvert < \left(\dfrac{1}{2}\right)^5 \cdot \dfrac{1}{10} = \dfrac{1}{320} < \dfrac{1}{200}$, since $\int_0^{1/2} e^{-t^2}\,dt = \sum\limits_{n=0}^{\infty} \dfrac{(-1)^n \left(\frac{1}{2}\right)^{2n+1}}{n!(2n+1)}$, which is an alternating series with individual terms that decrease in absolute value to 0.	$2: \begin{cases} 1: \text{uses the third term as the error bound} \\ 1: \text{explanation} \end{cases}$

CALCULUS BC
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

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5. A population is modeled by a function P that satisfies the logistic differential equation

$$\frac{dP}{dt} = \frac{P}{5}\left(1 - \frac{P}{12}\right).$$

(a) If $P(0) = 3$, what is $\lim_{t\to\infty} P(t)$?

If $P(0) = 20$, what is $\lim_{t\to\infty} P(t)$?

(b) If $P(0) = 3$, for what value of P is the population growing the fastest?

(c) A different population is modeled by a function Y that satisfies the separable differential equation

$$\frac{dY}{dt} = \frac{Y}{5}\left(1 - \frac{t}{12}\right).$$

Find $Y(t)$ if $Y(0) = 3$.

(d) For the function Y found in part (c), what is $\lim_{t\to\infty} Y(t)$?

Question 5

A population is modeled by a function P that satisfies the logistic differential equation

$$\frac{dP}{dt} = \frac{P}{5}\left(1 - \frac{P}{12}\right).$$

(a) If $P(0) = 3$, what is $\lim_{t\to\infty} P(t)$?

If $P(0) = 20$, what is $\lim_{t\to\infty} P(t)$?

(b) If $P(0) = 3$, for what value of P is the population growing the fastest?

(c) A different population is modeled by a function Y that satisfies the separable differential equation

$$\frac{dY}{dt} = \frac{Y}{5}\left(1 - \frac{t}{12}\right).$$

Find $Y(t)$ if $Y(0) = 3$.

(d) For the function Y found in part (c), what is $\lim_{t\to\infty} Y(t)$?

(a) For this logistic differential equation, the carrying capacity is 12. If $P(0) = 3$, $\lim_{t\to\infty} P(t) = 12$. If $P(0) = 20$, $\lim_{t\to\infty} P(t) = 12$.	$2: \begin{cases} 1: \text{answer} \\ 1: \text{answer} \end{cases}$
(b) The population is growing the fastest when P is half the carrying capacity. Therefore, P is growing the fastest when $P = 6$.	1 : answer
(c) $\frac{1}{Y}dY = \frac{1}{5}\left(1 - \frac{t}{12}\right)dt = \left(\frac{1}{5} - \frac{t}{60}\right)dt$ $\ln\lvert Y\rvert = \frac{t}{5} - \frac{t^2}{120} + C$ $Y(t) = Ke^{\frac{t}{5} - \frac{t^2}{120}}$ $K = 3$ $Y(t) = 3e^{\frac{t}{5} - \frac{t^2}{120}}$	$5: \begin{cases} 1: \text{separates variables} \\ 1: \text{antiderivatives} \\ 1: \text{constant of integration} \\ 1: \text{uses initial condition} \\ 1: \text{solves for } Y \\ \quad 0/1 \text{ if } Y \text{ is not exponential} \end{cases}$ Note: max $2/5$ [1-1-0-0-0] if no constant of integration Note: $0/5$ if no separation of variables
(d) $\lim_{t\to\infty} Y(t) = 0$	1 : answer $0/1$ if Y is not exponential